The Cycle Non Split Domination Number of An Intuitionistic Fuzzy Graph

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ABSTRACT

A dominating set D of an intuitionistic fuzzy graph G = (V, E) is a cycle non split dominating set if the induced intuitionistic fuzzy subgraph H = (V - D, V', E') is a cycle. The cycle non split domination number γcns (G) of G is the minimum intuitionistic fuzzy cardinality of a cycle non split dominating set. In this paper we study a cycle non split dominating sets of an intuitionistic fuzzy graphs and investigate the relationship of γcns (G) with other known parameters of G.

Keywords: Intuitionistic Fuzzy graph, Intuitionistic fuzzy domination, split domination number in IFG, Non split domination in IFG, cycle non split domination number in IFG.

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1. INTRODUCTION


2. PRELIMINARIES

Definition: 2.1
An Intuitionistic Fuzzy Graph (IFG) is of the form G = (V, E) where
(i) V = {V1, V2, ..., Vn} such that σ1 : V → [0, 1] and σ2 : V → [0, 1] denote the degree of membership and non-membership of the element vi ∈ V respectively and 0 ≤ σ1 (vi) + σ2 (vi) = 1, for every vi ∈ V
(ii) E ⊆ V × V where µ1 : V × V → [0, 1] and µ2 : V × V → [0, 1] are such that
µ1 (vi, vj) ≤ min {σ1(vi), σ1(vj)}
µ2 (vi, vj) ≤ max {σ2(vi), σ2(vj)}
and 0 ≤ µ1 (vi, vj) + µ2 (vi, vj) ≤ 1 for every (vi, vj) ∈ E.

Definition: 2.2
An IFG H = (V', E') is said to be an IF subgraph (IFSG) of G = (V, E) if V' ⊆ V and E' ⊆ E. That is σ1i ≤ σ1j ; σ2i ≥ σ2j and µ1i ≤ µ1j ; µ2i ≥ µ2j, for every i = 1, 2,..., n.

Definition: 2.3
The intuitionistic fuzzy subgraph H = (V’, E’) is said to be a spanning fuzzy subgraph of an IFG G = (V, E) if σ1 (u) = σ3 (u) and σ2 (u) = σ2 (u) for all u ∈ V’ and µ3(u,v) ≤ µ3(u,v) and µ3(u,v) ≥ µ2(u,v) for all u,v ∈ V.

Definition: 2.4
Let G = (V, E) be an IFG. Then the vertex cardinality of G is defined by
p = |V| = |Vi | + 1/2 (1 + σ1(vi) - σ2(vi)) for all vi ∈ V

Definition: 2.5
Let G = (V, E) be an IFG. Then the edge cardinality of E is defined by

Definition: 2.6
Let \( G = (V, E) \) be an IFG. Then the cardinality of \( G \) is defined to be \( |G| = |V| + |E| = p + q \).

Definition: 2.7
The number of vertices is called the order of an IFG and is denoted by \( O(G) \). The number of edges is called size of IFG and is denoted by \( S(G) \).

Definition: 2.8
The vertices \( v_i \) and \( v_j \) are said to be neighbors in IFG if either one of the following conditions hold
(i) \( \mu_1(v_i, v_j) > 0 \), \( \mu_2(v_i, v_j) > 0 \)
(ii) \( \mu_1(v_i, v_j) = 0 \), \( \mu_2(v_i, v_j) > 0 \)
(iii) \( \mu_1(v_i, v_j) > 0 \), \( \mu_2(v_i, v_j) = 0 \), \( v_i, v_j \in V \)

Definition: 2.9
A path in an IFG is a sequence of distinct vertices \( \ldots \) \( v_n \) such that either one of the following conditions is satisfied.
(i) \( \mu_1(v_i, v_j) > 0 \), \( \mu_2(v_i, v_j) > 0 \) for some \( i \) and \( j \)
(ii) \( \mu_1(v_i, v_j) = 0 \), \( \mu_2(v_i, v_j) > 0 \) for some \( i \) and \( j \)
(iii) \( \mu_1(v_i, v_j) > 0 \), \( \mu_2(v_i, v_j) = 0 \) for some \( i \) and \( j \)

Note: The length of the path \( P = v_1, v_2, \ldots v_{n+1} \) \( (n > 0) \) is \( n \).

Definition: 2.10
Two vertices that are joined by a path is called connected.

Definition: 2.11
An IFG \( G = (V, E) \) is said to be complete IFG if \( \mu_{1ij} = \min \{\sigma_1(v_i), \sigma_1(v_j)\} \) \( \mu_{2ij} = \max \{\sigma_2(v_i), \sigma_2(v_j)\} \) for all \( v_i, v_j \in V \).

Definition: 2.12
The complement of an IFG, \( G = (V, E) \) is an IFG, \( \overline{G} = (\overline{V}, \overline{E}) \), where
(i) \( \overline{V} = V \)
(ii) \( \overline{\sigma_{1i}} = \sigma_1 \) and \( \overline{\sigma_{2i}} = \sigma_2 \), for all \( i = 1, 2, \ldots n \)
(iii) \( \mu_{1ij} = \min \{\sigma_1(v_i), \sigma_1(v_j)\} - \mu_{1ij} \)
(iv) \( \mu_{2ij} = \max \{\sigma_2(v_i), \sigma_2(v_j)\} - \mu_{2ij} \) for all \( i = 1, 2, \ldots n \)

Definition: 2.13
An IFG, \( G = (V, E) \) is said to bipartite the vertex set \( V \) can be partitioned into two non empty sets \( V_1 \) and \( V_2 \) such that
(i) \( \mu_1(v_i, v_j) = 0 \) and \( \mu_2(v_i, v_j) = 0 \) if \( v_i, v_j \in V_1 \) (or)
\( v_i, v_j \in V_2 \) (ii) \( \mu_1(v_i, v_j) > 0 \) and \( \mu_2(v_i, v_j) > 0 \) if \( v_i \in V_1 \) and \( v_j \in V_2 \) for some \( i \) and \( j \)

Definition: 2.14
A bipartite IFG, \( G = (V, E) \) is said to be complete if
\( \mu_1(v_i, v_j) = \min \{\sigma(v_i), \sigma(v_j)\} \)
\( \mu_2(v_i, v_j) = \max \{\sigma(v_i), \sigma(v_j)\} \)

Definition: 2.15
A vertex \( u \in V \) of an IFG \( G = (V, E) \) is said to be an isolated vertex if \( \mu_1(u, v) = 0 \) and \( \mu_2(u, v) = 0 \), for all \( v \in V \). That is \( N(u) = \emptyset \). Thus an isolated vertex does not dominate any other vertex in \( G \).

Definition: 2.16
Let \( G = (V, E) \) be an IFG on \( V \). Let \( u, v \in V \), we say that \( u \) dominate \( v \) in \( G \) if there exits a strong arc between them.

Definition: 2.17
A subset \( D \) of \( V \) is called a dominating set in IFG \( G \) if for every \( v \in V - D \), there exists \( u \in D \) such that \( u \) dominates \( v \).

Definition: 2.18
A dominating set \( D \) of an IFG is said to be minimal dominating set if no proper subset of \( D \) is a dominating set.

Definition: 2.19
A vertex \( v \in G \) is said to be end - vertex of IFG if it has almost one strong neighbor in \( G \).

Definition: 2.20
A dominating set \( D \) of an intuitionistic fuzzy graph \( G = (V, E) \) is intuitionistic split dominating set if the induced fuzzy subgraph \( H = (V - D, \overline{V', E'}) \) is disconnected. The split domination number \( \gamma_s(G) \) of \( G \) is the minimum intuitionistic fuzzy cardinality of a split domination set.

Definition: 2.21
A dominating set \( D \) of an intuitionistic fuzzy graph \( G = (V, E) \) is intuitionistic fuzzy graph \( G = (V, E) \) is a non split dominating set if the induced intuitionistic fuzzy sub graph \( H = (V - D, \overline{V', E'}) \) is connected. The non split domination number \( \gamma_{ns}(D) \) of \( G \) is the minimum fuzzy cardinality of a non split domination set.
Definition: 2.22
A dominating set $D$ of an intuitionistic fuzzy graph $G = (V, E)$ is a cycle non split dominating set if the induced fuzzy subgraph $H = (\langle V - D \rangle, V', E')$ is a cycle. The cycle non split domination number $\gamma_{cns}(G)$ of $G$ is the minimum intuitionistic fuzzy cardinality of a cycle non split domination set.

Definition: 2.23
A dominating set $D$ of an intuitionistic fuzzy graph $G = (V, E)$ is a path non split dominating set if the induced fuzzy subgraph $H = (\langle V - D \rangle, V', E')$ is a path. The path non split domination number $\gamma_{pns}(G)$ of $G$ is the minimum intuitionistic fuzzy cardinality of a path non split domination set.

3. MAIN RESULTS

Theorem: 3.1
For any fuzzy intuitionistic fuzzy graph $G = (V, E)$, $\gamma(G) \leq \gamma_{cns}(G) \leq \gamma_{pns}(G)$.

Example:

$D = \{V_2, V_5\}$
$\gamma(G) = 0.85$
$D_{cns} = \{V_1, V_6\}$
$\gamma_{cns}(G) = 0.9$
$\gamma(G) \leq \gamma_{cns}(G)$

Proof:
Let $G = (V, E)$ be an intuitionistic fuzzy graph. Let $D$ be the minimum intuitionistic fuzzy dominating set. Let $D_1$ and $D_{cns}$ be the minimum intuitionistic fuzzy split dominating set and minimum intuitionistic fuzzy cycle non split dominating set of $G$ respectively. The cardinality of intuitionistic fuzzy dominating set need not exceed either one of the minimum cardinality of intuitionistic fuzzy split dominating set or intuitionistic fuzzy cycle non split dominating set.

Theorem 3.2.1
$\gamma(G) \leq \min \{\gamma_{s}(G), \gamma_{pns}(G)\}$

Theorem: 3.3
For any intuitionistic spanning fuzzy sub graph $H = (V', E')$ of IFG $G = (V, E)$, $\gamma_{cns}(H) \geq \gamma_{cns}(G)$.

Example: From Fig. (i)

$\gamma(G) = 0.85$, $\gamma_{s}(G) = 1.45$, $\gamma_{cns}(G) = 0.9$
$\gamma(G) \leq \min \{\gamma_{s}(G), \gamma_{cns}(G)\}$

Proof:
Let $G = (V, E)$ be an intuitionistic fuzzy graph. Let $D$ be the minimum intuitionistic fuzzy dominating set. Let $D_{cns}$ be the intuitionistic fuzzy cycle non split dominating set. $D_{cns}$ is also a dominating set but need not be minimum intuitionistic fuzzy dominating set. Therefore $|D| \leq |D_{cns}|$.

Clearly $\gamma(G) \leq \gamma_{cns}(G) \leq \gamma(G)$

Theorem 3.1.1
$\gamma(G) \leq \gamma_{pns}(G) \leq \gamma(G)$

Theorem: 3.2
For any intuitionistic fuzzy graph $G = (V, E)$, $\gamma(G) \leq \min \{\gamma_{s}(G), \gamma_{cns}(G)\}$. 

Example:

$\gamma_{cns}(G) = 0.9$
$D_{cns}(H) = \{V_1, V_6, V_8\}$
$\gamma_{cns}(H) = 1.3$
\[ 1.3 = \gamma_{cns}(H) \geq \gamma_{cns}(G) = 0.9 \]

**Proof:**
Let \( G = (V, E) \) be an intuitionistic fuzzy graph and let \( H = (V', E') \) be the intuitionistic spanning fuzzy sub graph of \( G \). \( D_{cns}(G) \) be the intuitionistic fuzzy minimum cycle non split dominating set of \( G \). \( D_{cns}(H) \) be the intuitionistic fuzzy cycle non split dominating set of \( H \) but not minimum.

\[ \gamma_{cns}(H) \geq \gamma_{cns}(G). \]

**Theorem: 3.3.1**
\[ \gamma_{pns}(H) \geq \gamma_{pns}(G). \]

**Theorem: 3.4**
For any intuitionistic complete fuzzy graph \( K_{\sigma, \mu} \),
\[ \gamma(G) = \gamma_{cns}(G) = \min \{|v_i| / v_i \in V\} \]

**Proof:**
Let \( G = (V, E) \) be a complete fuzzy graph therefore, there is a strong arc between every pair of vertices. We remove any vertex having minimum cardinality, the resulting graph is a cycle.

**Example:**

![Fig. (iv)](image)

Let \( D = \{v\} \) is minimum dominating set, then \( V - D \) is a cycle.

\[ \gamma(G) = \gamma_{cns}(G) = \min \{|v_i| / v_i \in V\}. \]

**Theorem: 3.5**
In an intuitionistic fuzzy graph \( G \), \( D_{cns} \) is the minimal cycle non split dominating set if and only if for each \( v \in D_{cns} \), one of the following two conditions holds.

(i) \( N(v) \cap D_{cns} = \phi \)

(ii) There is a vertex \( u \in V - D_{cns} \) such that \( N(u) \cap D_{cns} = \{v\} \).

**Proof:**
Let \( D_{cns} \) be a minimal cycle non split domination set of an intuitionistic fuzzy graph and \( v \in D_{cns} \), then \( D' = D_{cns} - \{v\} \) is not a cycle non split dominating set and hence there exist \( u \in V - D' \) such that \( u \) is not dominated by any element of \( D' \). If \( u = v \) we get (i) and if \( u \neq v \) we get (ii). The converse is obvious.

**Theorem: 3.5.1**
\( \gamma_{pns} \) set satisfies Ore's theorem.

**Theorem: 3.6**
Let \( G \) be a complete intuitionistic fuzzy graph \( K_{\sigma, \mu} \) then \( \gamma_{cns}(G) = \min \{|u|, \text{where} \ u \text{is the vertex having minimum intuitionistic fuzzy cardinality.} \}

Let \( G_i \) be a sub graph of \( G \) induced by \( V - u \) where \( u \) is the vertex of minimum cardinality, \( G_i \) has a vertex set \( V_i = \{V - u\} \) then \( \gamma_{cns}(G_i) = \gamma_{cns}(G_1) \cdot \gamma_{cns}(G_2) \cdot \cdots \cdot \gamma_{cns}(G_n) \) provided the intuitionistic fuzzy graph \( G_n \) is an elementary cycle with three vertices.

**Theorem: 3.7**
For any fuzzy graph without isolated vertices \( \gamma_{cns}(G) \leq p/2 \).

**Example:**

![Fig. (v)](image)

Proof:
Any graph without isolated vertices has two disjoint dominating sets and hence the result follows.

**Theorem: 3.8**
For any intuitionistic fuzzy graph, \( \gamma_{cns}(G) \leq p - \Delta_e \)

**Example:**
From Fig. (iv) \( p = 2.85, \Delta_e = 1.3, \gamma_{cns}(G) = 0.8 \)

**Proof:**
Let \( v \) be a vertex of an intuitionistic fuzzy graph, such that \( dN(v) = \Delta_e \), then \( V \setminus N(v) \) is a dominating set of \( G \), so that \( \gamma_{cns}(G) \cdot |V \setminus N(v)| = p - \Delta_e \).

**Theorem: 3.9**
For the domination number \( \gamma_{cns} \), the following theorem gives a Nordhaus - Gaddum type result.
For any intuitionistic fuzzy graph $G$, $\gamma_{cns}(G) + \gamma_{cns}(\overline{G}) \cdot 2p$.

**Proof:**
Let $G$ be a connected intuitionistic fuzzy graph it may or may not contain a cycle.

Suppose $G$ contains a cycle then by theorem 3.1, $\gamma_{cns}(G) \cdot p$.

Also $\overline{G}$ may or may not contain a cycle. We have $\gamma_{cns}(\overline{G}) \cdot p$ or $\gamma_{cns}(G) = 0$; vice versa.

Hence the inequality is trivial.

**Theorem 3.91**
$\gamma_{pns}(G) + \gamma_{pns}(\overline{G}) \cdot 2p$.

**REFERENCES**


