The Neighborhood Clique Domination Number in Fuzzy Graphs

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ABSTRACT

In this paper, dominating set D of a fuzzy graph $G=(\sigma,\mu)$ is a Neighborhood Clique dominating set if $< N(D_{nc}(G))>$ is complete provided N(D) contains the fuzzy vertices other than D. The fuzzy neighborhood clique domination number $\gamma_{nc}(G)$ is the minimum fuzzy cardinality taken over all minimal neighborhood clique dominating sets of G.

Keywords

Fuzzy graphs, Fuzzy domination, Clique domination number, Neighborhood clique domination number.

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1. INTRODUCTION

Kulli V.R. et.al introduced the concept of split domination and non-split neighborhood domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path,cycles and connectedness[9].A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs[10]. In this paper we discuss the Neighborhood Clique domination number of fuzzy graph and obtained the relationship with other known parametersof G.

2. PRELIMINARIES

Definition:2.1

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by $\gamma(G)$.

Definition: 2.2

Let $G=(\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all $u \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u,v\}) = \mu(\{u,v\})$ for all $u,v \in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition:.2.3

The fuzzy subgraph $H=(\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of $G=(\sigma, \mu)$ if $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and $\mu_1(u, v) \le \mu(u, v)$ for all $u,v \in V$. Let G (σ, μ) be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. ,$\sigma_1(u) \le \sigma(u)$ for all u.

Definition: 2.4

Let $G=(\sigma,\mu)$ be a fuzzy graph on V. Let $u, v \in V$. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u) \wedge \sigma(v)$. A subset D of V is called a dominating set in G if for every $v \notin D$, there exists $u \in D$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$or γ.

Definition: 2.5

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that

$$\sum_{v_i \in D'} \sigma(v_i) < \sum_{v_i \in D} \sigma(v_i)$$

Definition: 2.6

The order p and size q of a fuzzy graph $G=(\sigma,\mu)$ are defined to be $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(\{u,v\})$.

Definition: 2.7

An edge e={u, v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V / \mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v\in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u) / u\in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) / u\in V(G)\}$.

Definition: 2.8

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $v\in V-\{u\}$, that is, $N(u) =\phi$, Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.9

A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $u, v\in D$.

Definition: 2.10

The complement of a fuzzy graph G, denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma,\bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u)\wedge\sigma(v) - \mu(\{u,v\})$.

Definition: 2.11

Letσ: $V\rightarrow [0, 1]$ be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$ for all $uv \in E$ and is denoted by K_σ.

Definition: 2.12

A fuzzy graph $G=(\sigma,\mu)$ is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge\sigma(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.

Definition: 2.13

Let $G = (\sigma, \mu)$ be a regular fuzzy graph on $G^*= (V, E)$. If $d_G(v) = k$ for all $v\in V$, (i.e ,) if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where $G^*= (V, E)$ is an underlying crisp graph.

Remark: 2.14

G is k-regular graph iff $\delta=\Delta=k$.

Definition: 2.15

Let $G= (\sigma, \mu)$ be a fuzzy graph. The total degree of a vertex $u\in V$ is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv\in E} \mu(uv) +\sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph.

Definition: 2.16

A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by $\beta(G)$.

Definition: 2.17

Let $G = (\sigma, \mu)$ be a fuzzy graph on D and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e\in D} \mu(e)$.

Definition: 2.18

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v\in N(v)} \sigma(v)$ is called the neighborhood of u and is denoted by dN(u).

Definition: 2.19

The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max \{dE(u) \mid u \in V(G)\}$.

3. MAIN RESULTS

Definition 3.1

Let $G = (\sigma, \mu)$ be a fuzzy graph without isolated vertices. A subset $D_{nc}(G)$ of V is said to be a neighborhood clique dominating set if < N (D_{nc}(G))> is complete provided N(D) contains the fuzzy vertices other than D. The fuzzy neighborhood clique domination number γ_{nc}(G) is the minimum fuzzy cardinality taken over all minimal neighborhood clique dominating sets of G.

Example 3.2

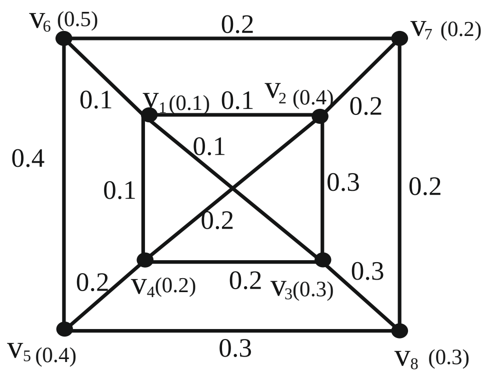

Fig. 1

$D_{nc}(G) = \{v_5, v_6, v_7, v_8\}$

$\gamma_{nc}(G) = 1.4$

<N (D_{nc}(G))> is complete

Theorem 3.3

If a fuzzy graph G = (σ, μ) is complete with σ(v_i) = c (constant) , for every $v_i \in V$ then $\gamma_{nc}(G) = c$.

Proof:

Let G = (σ, μ) be the complete fuzzy graph with vertex set V = {$v_1,v_2,\ldots v_{i-1},v_{i+1},\ldots,v_n$ },by the definition of fuzzy complete graph each v_i dominates the other vertices.

Let D_{nc}(G) be the neighborhood clique dominating set of G. D_{nc}(G) = {v_i / where v_i is the vertex of minimum fuzzy cardinality}.Therefore N(D_{nc}(G)) = { $v_1,v_2,\ldots v_i,v_{i+1},\ldots,v_n$ } and < N(D_{nc}(G)) > is complete. Since σ(v_i)'s are equa,l then the fuzzy neighborhood clique domination number $\gamma_{nc}(G) = \sigma(v_i) = c$.

Theorem 3.4

If a fuzzy graph G = (σ, μ) is complete and D_{nc}(G) is the neighborhood clique dominating set, then <V– D_{nc}(G) > is complete.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set V = { $v_1,v_2,\ldots v_i,v_{i+1},\ldots,v_n$ },by the definition of fuzzy complete graph each v_i isdominates the other vertices in G. The neighborhood clique dominating set D_{nc}(G) = { v_i / where v_i is the vertex of minimum fuzzy cardinality}. By definition of clique neighborhood dominating set <N (D_{nc}(G))> is complete. <V– D_{nc}(G) > = {$v_1, v_2, \ldots v_i, v_{i+1}, \ldots, v_n$ }, also <V– D_{nc}(G) > is complete.

Theorem 3.5

If a fuzzy graph G = (σ, μ) is complete and D_{nc}(G) is the neighborhood clique dominating set, then $\gamma_{nc}(G) = \min\{\sigma(v_i) / v_i \in V\}$.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set V = { $v_1,v_2,\ldots v_i,v_{i+1},\ldots,v_n$ } having distinct fuzzy vertex cardinality. Let D_{nc}(G) be the neighborhood clique dominating set of G. That is D_{nc} = {v_i / where v_i is the vertex of minimum fuzzy cardinality}. Therefore the neighborhood clique domination number $\gamma_{nc}(K_\sigma) = \min\{\sigma(v_i) / v_i \in V\}$.

Theorem 3.6

If a fuzzy graph G = (σ, μ) is complete and D_{nc}(G) is the neighborhood clique dominating set then $\gamma_{nc}(G) \le \gamma_{nc}(G_1) \le \gamma_{nc}(G_2) \le \gamma_{nc}(G_3) \le \ldots \le \gamma_{nc}(G_{n-1})$ where G_i is a fuzzy graph with V_i = { V_i – { v_i } / σ(v_i) = minimum fuzzy vertex cardinality }.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set V = { $v_1,v_2,\ldots v_i,v_{i+1},\ldots,v_n$ }. Let G_i be the fuzzy graph induced by the vertex set Vi = {V- D_{nc}(G) }. D_{nc}(G) is the neighborhood clique dominating set with respect to V_i , by definition < N(D_{nc}(G)) > is complete. Clear the neighborhood domination number of G_i's are γ_{nc} (G) , γ_{nc} (G_1), γ_{nc} (G_2), γ_{nc} (G_3), …,γ_{nc} (G_{n-1}) such that $\gamma_{nc}(G) \le \gamma_{nc}(G_1) \le \gamma_{nc}(G_2) \le \gamma_{nc}(G_3) \le \ldots \le \gamma_{nc}(G_{n-1})$.

Theorem 3.7

If G = (σ, μ) is a fuzzy graph, then $\gamma(G) \le \gamma_{nc}(G)$.

Proof:

Let G= (σ, μ) be the complete fuzzy graph. Let D(G) and D_{nc}(G) be the dominating set and neighborhood clique dominating set of G. Since $D_{nc}(G)$ be the neighborhood clique dominating set. $D_{nc}(G)$ is also dominating set, but need not be minimum fuzzy dominating set. Therefore $\gamma(G) \leq \gamma_{nc}(G)$.

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