The Cycle Non Split Domination Number in Fuzzy Graphs

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ABSTRACT

A dominating set D of a fuzzy graph G=(σ,μ) is a cycle non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ′,μ′) is a cycle. The cycle non split domination number $\gamma_{cns}(G)$ *of G is the minimum fuzzy cardinality of a cycle non split dominating set. In this paper we study a cycle non split dominating sets of fuzzy graphs and investigate the relationship of* $\gamma_{cns}(G)$ *with other known parameters of G.*

Keywords

Fuzzy graphs, Fuzzy domination, Non Split fuzzy domination number, Cycle non split domination number, Path non split domination number.

Subject Classification No. 05C72, 05C75

I. INTRODUCTION

Kulli V.R. et.al introduced the concept of split domination and non-split domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness [10]. A. Somasundram and S. Somasundram discussed domination in Fuzzy graphs [11]. Mahyoub Q.M. and Sonar N.D. discussed the split domination number of fuzzy graphs[6]. Ponnappan C.Y and et. al. discussed the strong non split domination number of fuzzy graphs [9]. In this paper we discuss the cycle non split domination number of fuzzy graph and obtained the relationship with other known parameters of G.

II. PRELIMINARIES

Definition: 2.1 [2]

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$.

Definition: 2.2 [3]

A dominating set D of a graph G=(V,E) is a split dominating set if the induced subgraph <V-D> is disconnected. The split domination number $\gamma_s(G)$ of a graph G is the minimum cardinality of a split dominating set.

Definition: 2.3 [3]

A dominating set D of a graph G=(V,E) is a non split dominating set if the induced subgraph <V-D> is connected. The non split domination number $\gamma_{ns}(G)$ of a graph G is the minimum cardinality of a non split dominating set.

Definition: 2.4 [4]

A dominating set D of a graph G=(V,E) is a cycle non split dominating set if the induced subgraph <V-D> is a cycle. The cycle non split domination number $\gamma_{cns}(G)$ of a graph G is the minimum cardinality of a cycle non split dominating set.

Definition: 2.5 [4]

A dominating set D of a graph G=(V,E) is a path non split dominating set if the induced subgraph <V-D>is a path. The path non split domination number $\gamma_{pns}(G)$ of a graph G is the minimum cardinality of a path non split dominating set.

Definition: 2.6 [10]

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=(σ,μ) is a set with two functions $\sigma : V\rightarrow[0,1]$ and $\mu: E\rightarrow[0,1]$ such that $\mu(\{u ,v\})\leq\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$.

Definition: 2.7 [11]

Let G=(σ,μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u,v\}) = \mu(\{u,v\})$ for all $u,v \in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition: 2.8 [11]

The fuzzy subgraph $H=(V_1,\sigma_1,\mu_1)$ is said to be a spanning fuzzy subgraph of G=(V,σ,μ) if $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) \le \mu(u,v)$ for all u,v∈V. Let G (V,σ,μ) be a fuzzy graph and μ_1 be any fuzzy subset of μ, i.e. , $\sigma_1(u) \le \sigma(u)$ for all u.

Definition: 2.9 [6]

A dominating set D of a fuzzy graph G=(σ,μ) is a split dominating set if the induced fuzzy subgraph H=(<V-D>,σ′,μ′) is disconnected.

The split domination number $\gamma_s(G)$ of G is the minimum fuzzy cardinality of a split dominating set.

Definition: 2.10 [6]

A dominating set D of a fuzzy graph G=(σ,μ) is a non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',μ') is connected.

The non split domination number $\gamma_{ns}(G)$ of G is the minimum fuzzy cardinality of a non split dominating set.

Definition: 2.11

A dominating set D of a fuzzy graph G=(σ,μ) is a cycle non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',μ') is a cycle.

The cycle non split domination number $\gamma_{cns}(G)$ is the minimum fuzzy cardinality of a cycle non split dominating set.

Definition: 2.12

A dominating set D of a fuzzy graph G=(σ,μ) is a path non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',μ') is a path.

The path non split domination number $\gamma_{pns}(G)$ is the minimum fuzzy cardinality of a path non split dominating set.

Definition: 2.13[11]

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p=\sum_{u \in V}\sigma(u)$ and $q=\sum_{(u,v) \in E}\mu(\{u,v\})$.

Definition: 2.14 [11]

An edge e={u ,v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v \in V / \mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u)|u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u)|u \in V(G)\}$.

Definition: 2.15 [11]

The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u) \wedge \sigma(v) - \mu(\{u,v\})$.

Definition: 2.16 [11]

Let σ:V→[0,1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u) \wedge \sigma(v)$ for all uv∈E and is denoted by K_σ.

Definition: 2.17 [11]

A fuzzy graph G=(σ,μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

Definition: 2.18 [11]

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that |D′|<|D|.

III. MAIN RESULTS

Proposition: 1

For any complete fuzzy graph K_σ then

$$\gamma(G) = \gamma_{cns}(G) = \min\{\sigma(u)/u \in V\}$$

Proposition: 2

For fuzzy bipartite graph K_{σ_1,σ_2},

$\gamma_{cns}(K_{\sigma_1,\sigma_2}) = \min\{\sigma(u)\} + \min\{\sigma(v)\}$, where $u \in V_1$ and $v \in V_2$

Proposition: 3

For fuzzy wheel $\gamma_{cns}(G)=\sigma(u)$ such that u is the spoke of the wheel.

Proposition: 4

$\gamma_{cns}(G \circ K_1)=\sum_i \sigma(u_i)$.where u_i is the pendant vertices of the corona and G contains at least one cycle.

Proposition: 5

The cycle non split dominating set exists for Petersen graph and Davidson graph.

Note:

The cycle non split dominating set does not exist for path, tree and fan.

Theorem: 1

For any fuzzy graph G=(σ,μ), $\gamma(G) \leq \gamma_{cns}(G)<p$

Proof

Let G=(σ,μ) be a fuzzy graph. Let D be the minimum dominating set. D_{cns} is the fuzzy cycle non split dominating set. D_{cns} is also a dominating set but need not be a minimum fuzzy dominating set.

Therefore we get $|D| \leq |D_{cns}|$

That is $\gamma(G) \leq \gamma_{cns}(G)$.

Example:

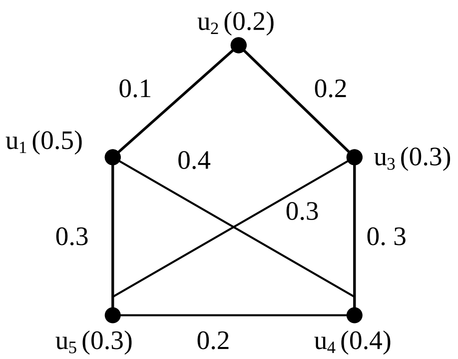

Fig. 1

$D=\{u_3,u_5\}$, $\gamma(G) = 0.6$

$D_{cns}=\{u_1,u_2\}$, $\gamma_{cns}(G) = 0.7$

Theorem 1.1

$\gamma(G) \leq \gamma_{pns}(G) < p$.

Theorem: 2

For any fuzzy graph G=(σ,μ),

$\gamma(G) \leq \min\{\gamma_s(G), \gamma_{cns}(G)\}$

Proof:

Let G=(σ,μ) be a fuzzy graph. D be the minimum fuzzy dominating set. Let D_s and D_{cns} the minimum fuzzy split dominating set and minimum fuzzy cycle non split dominating set of G respectively. The cardinality of fuzzy dominating set need not exceeds either one of the minimum of cardinality of fuzzy split dominating set or fuzzy cycle non split dominating set.

Therefore $|D| \leq \min\{|D_s|, |D_{cns}|\}$

Hence $\gamma(G) \leq \min\{\gamma_s(G), \gamma_{cns}(G)\}$

Example:

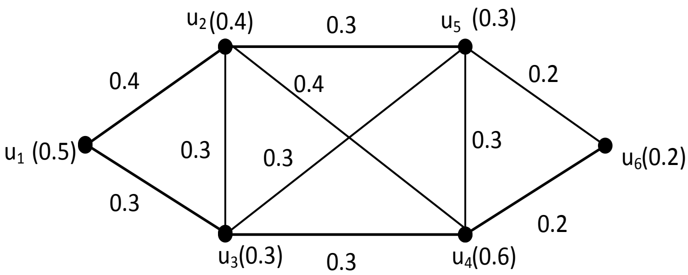

Fig. 2

Here D = $\{u_3,u_5\}$ $D_{cns} = \{u_1,u_6\}$, $D_s = \{u_2,u_3,u_4\}$

$\gamma(G) = 0.6, \gamma_{cns}(G) = 0.7, \gamma_s(G) = 1.3$

Theorem: 2.1

$\gamma(G) \leq \min\{\gamma_s(G), \gamma_{pns}(G)\}$

Theorem: 3

For any spanning fuzzy sub graph

$H = (\sigma', \mu')$ of G=(σ,μ),

$\gamma_{cns}(H) \geq \gamma_{cns}(G)$

Proof

Let G=(σ,μ) be a fuzzy graph and let H = (σ′,μ′) be the fuzzy spanning sub graph of G. $D_{cns}(G)$ be the fuzzy minimum cycle non-split dominating set of G. $D_{cns}(H)$ is fuzzy cycle non-split dominating set of H but not minimum.

Therefore, $\gamma_{cns}(H) \geq \gamma_{cns}(G)$.

Example:

Spanning fuzzy sub graph H of G.

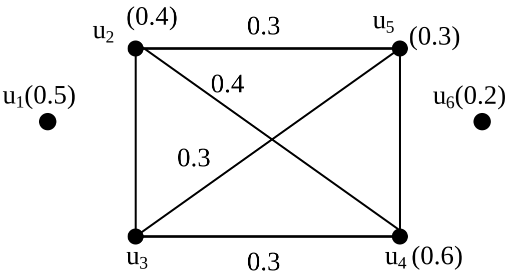

Fig. 3

$\gamma_{cns}(G) = 0.7, \quad \gamma_{cns}(H) = 1.0$

Theorem: 3.1

$\gamma_{pns}(H) \geq \gamma_{pns}(G)$.

Theorem: 4

Let G be a complete fuzzy graph $\mathbf{k}_\sigma$ then

$\gamma_{cns}(G) = \min\{\sigma(u)\}$, where u is the vertex having minimum cardinality.

Let G_i is subgraph of G induced by<V-u> where u is the vertex of minimum cardinality,G_i has a vertex set $V_i = \{V-u\}$ then

$\gamma_{cns}(G) \leq \gamma_{cns}(G_1) \leq \gamma_{cns}(G_2) \leq \ldots \leq \gamma_{cns}(G_n)$ provided the fuzzy graph G_n is a elementary cycle with three vertices.

Example:

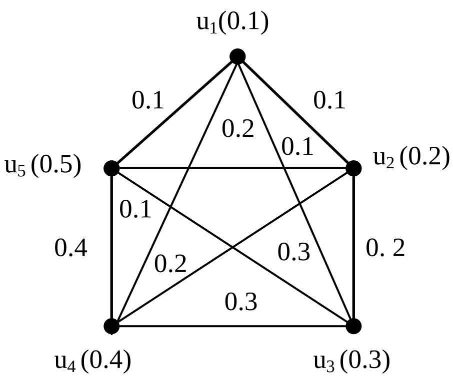

Fig. 4

$\gamma(G) = \gamma_{cns}(G) = 0.1$

G is a fuzzy graph induced by $\langle V-u_1 \rangle$

$\gamma_{cns}(G_1) = \sigma(u_2) = 0.2$

$\gamma_{cns}(G) \leq \gamma_{cns}(G_1)$.

Theorem: 5

For any fuzzy graph without isolated vertices

$\gamma_{cns}(G) \leq p/2$.

Proof:

Any graph without isolated vertices has two disjoint dominating sets and hence the result follows.

Example:

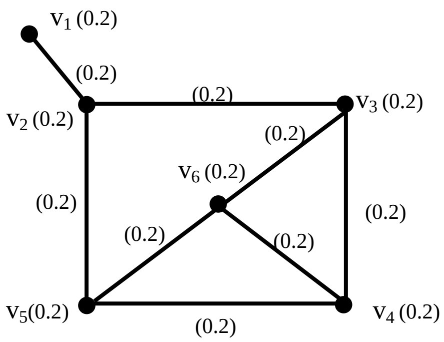

Fig. 5

$D_{cns}(G) = \{v_1, v_4\}$

$\langle V - D_{cns} \rangle$ is a cycle

p = 1.2, $\gamma_{cns}(G) = 0.4$

$\gamma_{cns}(G) \leq p/2$

Theorem: 6

For any fuzzy graph, $\gamma_{cns}(G) \leq p - \Delta_E$

Proof:

Let v be a vertex of a fuzzy graph, such that

$dN(v) = \Delta E$, then $V \setminus N(v)$ is a dominating set of G, so that

$\gamma_{cns}(G) \leq |V \setminus N(v)| = P - \Delta_E$.

Example:

From fig. 5, $p = 1.2$, $\Delta_E = 0.6$, γ_{cns} *(G)=0.4*

Theorem: 7

For any non trivial connected fuzzy graph G,

$\gamma(G) + \gamma_{pns}(G) \leq p$ and this bound this sharp, the path P_4 and cycle C_4 achieve this bound.

Theorem: 8

A cycle non split dominating set D of G=(σ,μ) is minimal if and only if for each v∈D one of the following two conditions holds

(i) $N(v) \cap D_{cns} = \varphi$

(ii) there is a vertex $u \in V\text{-}D_{cns}$

such that $N(u) \cap D_{cns} = \{v\}$

Proof:

Let D be a minimal cycle non split dominating set and v∈D, then D′=D-{v} is not a cycle non-split dominating set and hence there exist u∈V-D′ such that u is not dominated by any element of D′. If u=v we get (i) and if u≠v we get (ii). The converse is obvious.

Theorem: 9

For the domination number γ_{cns} the following theorem gives a Nordhaus – Gaddum type result.

For any fuzzy graph G, $\gamma_{cns}(G) + \gamma_{cns}(\bar{G}) \leq 2p$.

Proof:

Let G be a connected fuzzy graph it may or may not contains a cycle.

Suppose G contains a cycle then by theorem $\gamma_{cns}(G) \leq p$.

Also $\bar{G}$ may or may not contain a cycle. We have $\gamma_{cns}(\bar{G}) \leq p$ or $\gamma_{cns}(\bar{G}) = 0$ vice versa.

Hence the inequality is trivial.

Theorem: 10

For any complete fuzzy graph K_σ with $n \geq 4$ fuzzy vertices, $\gamma_{cns}(G) = \sum_{i=1}^{n-3} \sigma(v_i)$, where $\sigma(v_1) \leq \sigma(v_2) \leq \sigma(v_3) \leq, \ldots, \leq \sigma(v_{n-3})$.

Proof:

Let K_σ be the complete fuzzy graph with n vertices$\{v_1, v_2, v_3,\ldots,v_{n-1}, v_n\}$ such that v_i is adjacent with all other vertices$\{v_1, v_2, v_3,\ldots,v_{i-1},v_{i+1},\ldots,v_n\}$, further by definition of complete fuzzy graph, each v_i dominates the all other vertices. Let D_{cns} be the cycle non split dominating set. Therefore $D_{cns} = \{ v_i \ / \ i= 1$ to n-3, v_i are first n–3 minimum fuzzy vertex cardinality }such that $\langle V-D_{cs}\rangle$ is cycle and $\gamma_{cns}(G) = \sum_{i=1}^{n-3} \sigma(v_i)$.

Theorem: 11

For fuzzy complete bipartite graph K_{σ_1,σ_2}, $\gamma_{cns}(K_{\sigma_1,\sigma_2}) = \sum_{i=1}^{m-2} \sigma(vi) + \sum_{i=1}^{n-2} \sigma(vj)$ where $v_i \in V_1$ and $v_j \in V_2$ with $\sigma(v_1) \leq \sigma(v_2) \leq \sigma(v_3) \leq \ldots \leq \sigma(v_{m-2})$ and $\sigma(v_1) \leq \sigma(v_2) \leq \sigma(v_3) \leq \ldots \leq \sigma(v_{n-2})$.

Proof:

Let K_{σ_1,σ_2} be a fuzzy complete bipartite graph. By definition V can be partitioned into two sets V_1 and V_2. Let $V_1=\{u_1, u_2, u_3,\ldots, u_m\}$ and $V_2 =\{ v_1, v_2, v_3,\ldots, v_n\}$ with $4 \leq m \leq n$. u_i is adjacent with v_j, i= 1,2,…, m and j= 1,2,…,n, also u_i dominates v_j, i= 1,2,…, m and j= 1,2,…, n. Let D_{cns} be the fuzzy Cycle non split dominating set , therefore $D_{cns}= \{ v_i , v_j \ / \ i= 1$ to m–2; j= 1 to n–2 ; $v_i \in V_1$ are the vertices of first (m–2) minimum fuzzy cardinality and $v_j \in V_2$ are the vertices of first (n–2) minimum fuzzy cardinality} such that $\langle V-D_{cns}\rangle$ is a cycle. γ_{cns} is the minimum fuzzy cardinality of the cycle non split dominating set.

Therefore $\gamma_{cns}(K_{\sigma_1,\sigma_2}) = \sum_{i=1}^{m-2} \sigma(vi) + \sum_{i=1}^{n-2} \sigma(vj)$.

Theorem: 12

$\gamma_{cns}(W_{n+1}) = \sigma(v)$, Where v is the centre of the fuzzy wheel.

Proof:

Let W_{n+1} be the fuzzy wheel with vertex set $\{ v, v_1, v_2, v_3,\ldots,v_{n-1}, v_n \}$such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$, $1 \leq i \leq n$, v adjacent with v_i , i=1,2,..,n. Clearly v_i is dominated by $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$, $1 \leq i \leq n$ and v dominates v_i , i=1,2,..,n. Let D_{cns} be the cycle non split dominating set.(i. e,) $D_{cns} = \{v\}$ such that $\langle V- D_{cns}\rangle$ is a cycle. γ_{cns} is the minimum fuzzy cardinality of the cycle non split dominating set. Obviously $\gamma_{cns}(W_{n+1}) = \sigma(v)$.

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