The Perfect Disconnected Domination Number in Fuzzy Graphs

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ABSTRACT

A dominating set D of a fuzzy graph G=(σ,μ) is a D_{pd}(G)of V is said to be a perfect disconnected dominating set if D_{pd}(G) is perfect and < D_{pd}(G)> is disconnected. The fuzzy perfect disconnected domination number γ_{pd} (G) is the minimum fuzzy cardinality taken over all minimal perfect disconnected dominating sets of G.

Keywords

Fuzzy graphs, Fuzzy domination, Connected fuzzy domination number, Disconnected fuzzy domination number, perfect domination number, Perfect disconnected domination number.

Subject Classification No. 05C72, 05C75

1. INTRODUCTION

Kulli V.R. et.al introduced the concept of connected domination and disconnected domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness [10]. A. Somasundram and S. Somasundram discussed domination in Fuzzy graphs [11]. In this paper we discuss the perfect disconnected domination number of fuzzy graph and obtained the relationship with other known parameters of G.

2. PRELIMINARIES

Definition: 2.1

Let G= (V, E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by γ(G).

Definition: 2.2

Let G=(σ, μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all $u\in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all $u,v \in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition: 2.3

The fuzzy subgraph $H=(\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of G=(σ, μ) if $\sigma_1(u)=\sigma(u)$ for all $u\in V_1$ and $\mu_1(u, v)\leq\mu(u, v)$ for all $u,v\in V$. Let G (σ, μ) be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. ,$\sigma_1(u)\leq\sigma (u)$ for all u.

Definition: 2.4

Let G=(σ,μ) be a fuzzy graph on V. Let u, v∈V. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every $v\notin D$, there exists u∈D such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G)or γ.

Definition: 2.5

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that

$$\sum_{v_i \in D'} \sigma(v_i) < \sum_{v_i \in D} \sigma(v_i)$$

Definition: 2.6

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p = \sum_{u\in V}\sigma(u)$ and $q = \sum_{(u,v)\in E} \mu(\{u ,v\})$.

Definition: 2.7

An edge e={u, v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v \in V / \mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u) / u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) / u \in V(G)\}$.

Definition: 2.8

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $v \in V-\{u\}$, that is, $N(u) = \phi$, Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.9

A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $u, v \in D$.

Definition: 2.10

The complement of a fuzzy graph G, denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u) \wedge \sigma(v) - \mu(\{u,v\})$.

Definition: 2.11

Let $\sigma: V \rightarrow [0, 1]$ be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u) \wedge \sigma(v)$ for all $uv \in E$ and is denoted by K_σ.

Definition: 2.12

A fuzzy graph $G=(\sigma,\mu)$ is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.

Definition: 2.13

Let $G = (\sigma, \mu)$ be a regular fuzzy graph on $G^* = (V, E)$. If $d_G(v) = k$ for all $v \in V$, i.e. if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where $G^* = (V, E)$ is an underlying crisp graph.

Remark: 2.14

G is k-regular graph iff $\delta=\Delta=k$.

Definition: 2.15

Let $G= (\sigma, \mu)$ be a fuzzy graph. The total degree of a vertex $u \in V$ is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv \in E} \mu(uv) + \sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph.

Definition: 2.16

A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by $\beta(G)$.

Definition: 2.17

Let $G = (\sigma, \mu)$ be a fuzzy graph on D and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.

Definition: 2.18

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v \in N(v)} \sigma(v)$ is called the neighborhood of u and is denoted by dN(u).

Definition: 2.19

The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max \{dE(u) \mid u \in V(G)\}$.

3. MAIN RESULTS

Definition 3.1

Let $G = (\sigma, \mu)$ be a fuzzy graph without isolated vertices. A subset $D_{pd}(G)$of V is said to be a perfect disconnected dominating set if $D_{pd}(G)$ is perfect and $< D_{pd}(G) >$ is disconnected. The fuzzy perfect disconnected domination number $\gamma_{pd}(G)$ is the minimum fuzzy cardinality taken over all minimal perfect disconnected dominating sets of G.

Example 3.2

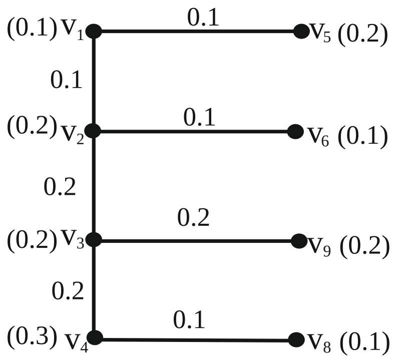

Fig. 1

$D_{pd}(G) = \{v_5, v_6, v_7, v_8\}$

$<D_{pd}(G)>$ is disconnected

$\gamma_{pd}(G) = 0.6$

Theorem 3.3

If G = (σ, μ) is a fuzzy graph then $\gamma_{pd}(G) < \frac{p}{2}$.

Proof:

Let G = (σ, μ) be a fuzzy graph. The fuzzy perfect disconnected dominating set $D_{pd}(G) = \{v_i / v_i \in G\}$ such that every vertex in V is exactly dominated by one vertex in D and $< D_{pd}(G)>$ is disconnected. Therefore, the fuzzy perfect disconnected domination number $\gamma_{pd}(G) < \frac{p}{2}$. The minimum cardinality of the fuzzy disconnected domination number $\gamma_{pd}(G) < \frac{p}{2}$.

Corollary 3.4

If G = (σ, μ) be a fuzzy graph with $\sigma(v_i) = c$, for every $v_i \in V$ then $\gamma_{pd}(G) = \frac{p}{2}$.

Theorem 3.5

If G = (σ, μ) is a fuzzy graph and has a $\gamma_{pc}(G)$ - set and $\gamma_{pd}(G)$ - set then $\gamma_{pc}(G) + \gamma_{pd}(G) = p$ where $\gamma_{pc}(G)$ is a fuzzy connected perfect domination number of G.

Proof:

Let G = (σ, μ) be a fuzzy graph and G has a $\gamma_{pc}(G)$ - set and $\gamma_{pd}(G)$ - set then by definition of perfect connected and perfect disconnected dominating set $\gamma_{pc}(G)$ has $\frac{n}{2}$ fuzzy vertices and $\gamma_{pd}(G)$ has $\frac{n}{2}$ fuzzy vertices, further $|D_{pc}(G)| = \frac{n}{2}$ and $|D_{pd}(G)| = \frac{n}{2}$, therefore $\gamma_{pc}(G) = \frac{p}{2}$, $\gamma_{pd}(G) = \frac{p}{2}$. Therefore, $\gamma_{pc}(G) + \gamma_{pd}(G) = p$.

Theorem 3.6

If G = (σ, μ) is a fuzzy graph and has $\gamma_{pd}(G)$ - set then the number of fuzzy vertices are even.

Proof:

If G = (σ, μ) is a fuzzy graph with n vertices and let $D_{pd}(G)$ be the fuzzy perfect disconnected dominating set, then by definition of perfect dominating set, every v in V is dominated by exactly one vertex in $D_{pd}(G)$. Therefore $|D_{pd}(G)|$ has even or odd number of fuzzy vertices. By definition of perfect disconnected dominating set V has even number of fuzzy vertices.

Theorem 3.7

If G = (σ, μ) is a fuzzy graph then $\gamma(G) \leq \gamma_t(G) \leq \gamma_p(G) \leq \gamma_{pd}(G)$ where $\gamma_t(G)$ and $\gamma_p(G)$ are the fuzzy total and perfect domination numbers respectively.

Proof:

Let G = (σ, μ) be a fuzzy graph and D, D_t, D_p and D_{pd} be the fuzzy dominating, fuzzy total dominating, fuzzy perfect dominating and fuzzy perfect disconnected dominating sets of G respectively. Since every total dominating set is a dominating set. Therefore $\gamma(G) \leq \gamma_t(G)$ and every perfect dominating set is total dominating set then $\gamma_t(G) \leq \gamma_p(G)$. Moreover, every perfect dominating set, $\gamma_p(G) \leq \gamma_{pd}(G)$. Finally, we have $\gamma(G) \leq \gamma_t(G) \leq \gamma_p(G) \leq \gamma_{pd}(G)$.

Theorem 3.8

If G = (σ, μ) is $K_n \circ K_1$ then $\gamma_{pc}(K_n \circ K_1) + \gamma_{pd}(K_n \circ K_1) = p$.

Proof:

Let G = (σ, μ) be a fuzzy Carona K_n o K_1 with vertex set $V = \{ v_1, v_2, \ldots v_n, v_{n+1}, \ldots, v_{2n} \}$, by definition the perfect disconnected dominating set $D_{pd}(G)$ has all fuzzy pendent vertices of G. Further $<D_{pd}(G)>$ is disconnected and $\{V - D_{spd}(G)\}$ form a γ_{pc} (G) - set, since $<V - D_{spd}(G)>$ is connected, obviously $\gamma_{pc}(K_n \text{ o } K_1) + \gamma_{pd}(K_n \text{ o } K_1) = p$.

Corollary: 3.9

If G = (σ, μ) is a fuzzy carona K_n o K_1 then γ_{pd} (G) is the inverse domination number of $\gamma_{pc}(G)$, also $\gamma_{pc}(K_n \text{ o } K_1) + \gamma_{pd}(K_n \text{ o } K_1) = p$.

Theorem 3.10

If G = (σ, μ) is K_n o K_1 and $\sigma(v_i) = c$ then $\gamma_{cr}(G) + \gamma_{pd}(G) = 2\sum_{i=1}^{n} \sigma(v_i)$, where $v_i \in V$ are the fuzzy pendent vertices of G.

Proof:

Let G = (σ, μ) be a fuzzy carona K_n o K_1 and D_{cr} (G), D_{pd} (G) be the fuzzy clique regular, perfect disconnected dominating sets respectively, then by definition of fuzzy clique regular dominating set $< N(D_{rc}(G)) >$ is regular, therefore D_{cr} (G) contains the pendent vertices of G and by definition of fuzzy perfect connected dominating set $< D_{pd}(G) >$ is disconnected, the fuzzy clique regular domination number and fuzzy perfect disconnected domination numbers are $\gamma_{cr}(G)$, γ_{pd} (G). Clearly, $\gamma_{cr}(G) + \gamma_{pd}(G) = 2\sum_{i=1}^{n} \sigma(v_i)$, where vi $\in V$ are the fuzzy pendent vertices of G.

Corollary: 3.11

If G = (σ, μ) is C_n o K_1 and $\sigma(v_i)$'s are equal with effective edges then $\gamma_{cr}(G) + \gamma_{pd}(G) = p$.

Corollary: 3.12

If G = (σ, μ) is K_n o K_1 and $\sigma(v_i)$'s are equal with effective edges then $\gamma_{cr}(G) + \gamma_{pd}(G) = p$.

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