The Clique Regular Domination Number in Fuzzy Graphs

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ABSTRACT

A dominating set D of a fuzzy graph G=(σ,μ) is a a clique regular dominating set if < N (D_{cr}(G)) > is regular. The fuzzy clique regular domination number $\gamma_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal clique regular dominating sets of G

Keywords

Fuzzy graphs, Fuzzy domination, Clique domination number, Clique regular domination.

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1. INTRODUCTION

Kulli V.R. et.alintroduced the concept of regular domination and Clique domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness[9]. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs[10]. In this paper we discuss the clique regular domination number in fuzzy graphs and obtained the relationship with other known parameters of G.

2. PRELIMINARIES

Definition:2.1

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by γ(G).

Definition: 2.2

Let G=(σ, μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$for all $u\in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all u,v $\in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $\langle V_1\rangle$.

Definition:.2.3

The fuzzy subgraph $H=(\sigma_1, \mu_1)$ is said to be a spanning fuzzy subgraph of G=(σ, μ) if $\sigma_1(u)=\sigma(u)$ for all $u\in V_1$ and $\mu_1(u, v)\leq\mu(u, v)$ for all u,v∈V. Let G (σ, μ) be a fuzzy graph and σ_1 be any fuzzy subset of V_1, i.e. ,$\sigma_1(u)\leq\sigma (u)$ for all u.

Definition: 2.4

Let G=(σ,μ) be a fuzzy graph on V. Let u, v∈V. We say that u dominates v in G if $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$. A subset D of V is called a dominating set in G if for every $v\notin D$, there exists u∈D such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G)or γ.

Definition: 2.5

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that

$$\sum_{v_i \in D'} \sigma(v_i) < \sum_{v_i \in D} \sigma(v_i)$$

Definition: 2.6

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p = \sum_{u\in V}\sigma(u)$ and $q = \sum_{(u,v)\in E} \mu(\{u ,v\})$.

Definition: 2.7

An edge e={u, v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V/ \mu(\{u ,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v\in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u) / u\in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) / u\in V(G)\}$.

Definition: 2.8

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(\{u ,v\}) < \sigma(u) \wedge \sigma(v)\}$ for all $v\in V-\{u\}$, that is, $N(u) =\phi$, Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.9

A set D of vertices of a fuzzy graph is said to be independent if $\mu(\{u,v\}) < \sigma(u) \wedge \sigma(v)\}$for all u , $v\in D$.

Definition: 2.10

The complement of a fuzzy graph G, denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma,\bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u)\wedge\sigma(v) - \mu(\{u,v\})$.

Definition: 2.11

Letσ: V→ [0, 1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$ for all uv $\in E$ and is denoted by K_σ.

Definition: 2.12

A fuzzy graph $G=(\sigma,\mu)$ is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further, if $\mu(u,v)=\sigma(u) \wedge\sigma(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are the restrictions of σ to V_1 and V_2 respectively.

Definition: 2.13

Let $G = (\sigma, \mu)$ be a regular fuzzy graph on $G^*= (V, E)$. If $d_G(v) = k$ for all $v\in V$, (i.e ,) if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or k-regular fuzzy graph. Where $G^*= (V, E)$ is an underlying crisp graph.

Remark: 2.14

G is k-regular graph iff $\delta=\Delta=k$.

Definition: 2.15

Let $G= (\sigma, \mu)$ be a fuzzy graph. The total degree of a vertex $u\in V$ is defined by $td_G(u) = d_G(u) + \sigma(u) = \sum_{uv\in E}\mu(uv) +\sigma(u)$. If each vertex of G has the same total degree k then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph.

Definition: 2.16

A set of fuzzy vertex which covers all the fuzzy edges is called a fuzzy vertex cover of G and the minimum cardinality of a fuzzy vertex cover is called a vertex covering number of G and denoted by $\beta(G)$.

Definition: 2.17

Let $G = (\sigma, \mu)$ be a fuzzy graph on D and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e\in D} \mu(e)$.

Definition: 2.18

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident of 'u' and is denoted by dE(u). $\sum_{v\in N(v)} \sigma(v)$ is called the neighborhood of u and is denoted by dN(u).

Definition: 2.19

The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max \{dE(u) \mid u \in V(G)\}$.

3. MAIN RESULTS

Definition 3.1

Let $G = (\sigma, \mu)$ be a fuzzy graph without isolated vertices. A subset $D_{cr}(G)$of V is said to be a clique regular dominating set if < N ($D_{cr}(G)$) > is regular. The fuzzy clique regular domination number $\gamma_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal clique regular dominating sets of G

Example 3.2

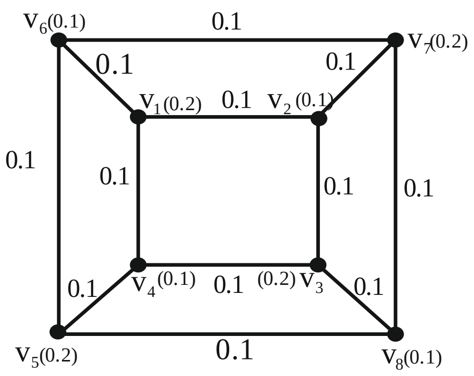

Fig. 1

$D_{cr}(G) = \{v_1, v_2, v_3, v_4\}$

$\gamma_{cr}(G) = 0.4$

$< N(D_{cr}(G) >$ is regular.

Theorem 3.3

If G = (σ, μ) is a regular fuzzy graph, then $\gamma_{cr}(G)$ - set exists.

Proof:

Let G = (σ, μ) be the complete fuzzy graph with n vertices set $\{ v_1, v_2, \ldots . v_i, v_{i+1}, \ldots, v_n \}$. Let $D_{cr}(G)$ be a clique regular dominating set which is a subset of a fuzzy vertex set of V of G. Since G is a regular then d(v) = k, for some k ∊ [0, 1], for every $v_i \in V$. The induced graphs of $N(D_{cr}(G))$ other than $D_{cr}(G)$ is regular. Therefore, $\gamma_{cr}(G)$ - set exists.

Theorem 3.4

If a fuzzy graph G = (σ, μ) is complete with $\sigma(v_i) = c$ (constant), for every $v_i \in V$, then $\gamma_{cr}(G) = c$.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set $V = \{ v_1, v_2, \ldots . v_i, v_{i+1}, \ldots, v_n \}$ and all vertices having constant fuzzy vertex cardinality. Therefore, each vertex has degree (n -1) $\sigma(v_i)$, clearly G is a regular fuzzy graph. Let $D_{cr}(G)$ be a clique regular fuzzy graph, that is $D_{cr}(G) = \{ v_i$ / for any $v_i \in V \}$, by definition $< N(D_{cr}(G)) >$ is regular. Therefore $\gamma_{cr}(G)$ is the fuzzy clique regular domination number of G, which is equal to $\sigma(v_i) = c$, that is $\gamma_{cr}(G) = \sigma(v_i) = c$.

Corollary: 3.5

If G = (σ, μ) is a totally regular fuzzy graph and $\gamma_{cr}(G)$ - set exists, then the alternate fuzzy edges having equal fuzzy cardinality (or) all the fuzzy vertices having equal fuzzy vertex cardinality.

Theorem 3.6

Every fuzzy complete graph $G = K_{\sigma}$, n ≥ 2 has a $\gamma_{cr}(G)$ - set with $\sigma(v_i) = c$ (constant) , for every $v_i \in V$.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set $V = \{ v_1, v_2, \ldots . v_i, v_{i+1}, \ldots, v_n \}$ and all vertices having constant fuzzy vertex cardinality. Moreover, every vertex in V dominates the other and having $\sigma(v_i) = c$, for every $v_i \in V$. Let $D_{cr}(G)$ be a clique regular fuzzy graph, that is $D_{cr}(G) = \{ v_i$ / for any $v_i \in V \}$, by definition $V - D_{cr}(G) = \{ v_1, v_2, \ldots . v_{i-1}, v_{i+1}, \ldots, v_n \}$ and $< V - D_{cr}(G) >$ is a regular graph with degree (n -2)c. Proceeding like this way, the complete fuzzy graph with n≥ 2 has a $\gamma_{cr}(G)$ - set.

Theorem 3.7

Every fuzzy wheel $G = W_{n+1}$ with $\sigma(v_i) = c$ (constant) for every $v_i \in V$ has a $\gamma_{cr}(G)$ - set.

Proof:

Let G= (σ, μ) be the fuzzy wheel W_{n+1} with vertex set $V = \{ v, v_1, v_2, \ldots . v_i, v_{i+1}, \ldots, v_n \}$ and all vertices having constant fuzzy vertex cardinality. The clique regular dominating set $D_{cr}(G) = \{ v$ / where v is the centre of the wheel such that $< N(D_{cr}(G)) >$ is a regular fuzzy graph}. Thus, every fuzzy wheel G with constant fuzzy vertex cardinality has a $\gamma_{cr}(G)$ - set.

Theorem 3.8

Every fuzzy complete bipartite graph has a $\gamma_{cr}(G)$ - set with $\sigma(v_i) = c$ (constant) for every $v_i \in V$ then $\gamma_{cr}(G) = 2c$.

Proof:

Let G= (σ, μ) be the fuzzy complete bipartite graph K_{σ_1, σ_2} with the vertex set $V = U \cup W$, $U \cap W = \phi$, $|U| = |W| = n$ and every vertex in V has a constant fuzzy vertex cardinality . Let $D_{cr}(G)$ be the clique regular dominating set which contains the vertices u_i and w_i, where $u_i \in U$ and $w_i \in W$, i = 1 to n. < $N(D_{cr}(G))$ > is regular with degree (n- 1)c. Therefore, $\gamma_{cr}(G)$ is the fuzzy clique regular domination number which is equal to 2c. That is $\gamma_{cr}(G) = 2c$.

Theorem 3.9

If a fuzzy graph G = (σ, μ) is complete bipartite K_{σ_1,σ_2} with distinct fuzzy vertex cardinality, then $\gamma_{cr}(G) = \min \{ \sigma(u_i) \} + \min \{ \sigma(v_i) \}$ where $u_i \in V_1$ and $v_i \in V_2$ with $V = V_1 \cup V_2$

Proof:

Let G= (σ, μ) be the fuzzy complete bipartite graph K_{σ_1,σ_2} with the vertex set $V = V_1 \cup V_2$ and $|V_1| = m$, $|V_2| = n$ all vertices having distinct fuzzy vertex cardinality. The clique regular dominating set $D_{cr}(G) = \{u, v \ / \ u \in V_1, v \in V_2$ and $\sigma(u) + \sigma(v)$ is minimum such that < $N(D_{cr}(G))$ > is regular }. Therefore the fuzzy clique regular domination number $\gamma_{cr}(G) = \min \{ \sigma(u_i) \} + \min \{ \sigma(v_i) \}$ where $u_i \in U$ and $v_i \in V$ with $V = V_1 \cup V_2$

Theorem 3.10

If G = (σ, μ) is a complete fuzzy graph with equal fuzzy cardinality then $\gamma_{nc}(G) = \gamma_{cr}(G)$.

Proof:

Let G= (σ, μ) be the complete fuzzy graph with vertex set V = { $v_1,v_2,\ldots.v_i,v_{i+1},\ldots,v_n$ } and all vertices having constant fuzzy vertex cardinality. Let $D_{cr}(G)$, $D_{nc}(G)$ be the fuzzy clique regular and neighborhood clique dominating sets respectively. By definition The clique regular dominating set < $N(D_{cr}(G))$ > is a regular and neighborhood clique dominating set < $N(D_{cr}(G))$ > is complete. By definition of complete fuzzy graph $D_{cr}(G) = D_{nc}(G)$ ={ vi / vi is the vertex of minimum fuzzy cardinality}. Moreover, $\gamma_{nc}(G)$, $\gamma_{cr}(G)$ are the fuzzy neighborhood domination and clique regular domination numbers respectively. Therefore, $\gamma_{nc}(G) = \gamma_{cr}(G)$.

Theorem 3.11

If G = (σ, μ) is a fuzzy cycle then $\gamma_{cr}(G) = p - \max \{ \sigma(v_i) + \sigma(v_{i+1}) \}$ where v_i and v_{i+1} are adjacent fuzzy with all effective edges.

Proof:

Let G = (σ, μ) be a fuzzy cycle graph with vertex set V = { $v_1,v_2,\ldots.v_i,v_{i+1},\ldots,v_n$ } such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \le i \le n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$. Let $D_{cr}(G)$ be the clique regular dominating set of G. $D_{cr}(G) = \{ V - \{v_i,v_{i+1}\} \ / \ v_i,v_{i+1}$ are adjacent fuzzy vertices with $\sigma(v_i) = \sigma(v_{i+1})$ then $\sigma(v_i) + \sigma(v_{i+1})$ is maximum}. Such that < N ($D_{cr}(G)$) > is regular and $\gamma_{cr}(G)$ is the minimum fuzzy cardinality taken over all minimal clique regular dominating sets of G. Therefore, $\gamma_{cr}(G) = p - \max\{ \sigma(v_i) + \sigma(v_{i+1}) \}$.

Theorem 3.12

If G = (σ, μ) is a fuzzy cycle with equal fuzzy vertex cardinality, then $\gamma_{cr}(G) = p - 2c$.

Proof:

Let G = (σ, μ) be a fuzzy cyclic graph with vertex set V = { $v_1,v_2,\ldots.v_i,v_{i+1},\ldots,v_n$ } such that v_i is adjacent to $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \le i \le n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$, given $\sigma(v_i) = c$, $v_i \in G$. Let $D_{cr}(G)$ be the clique regular dominating set, then by theorem 2.3.11, $\gamma_{cr}(G) = p - \max \{ \sigma(v_i) + \sigma(v_{i+1}) \}$, since $\sigma(v_i)$'s are equal max { c + c} = 2c. Further, $\gamma_{cr}(G) = p - 2c$, that is, $\gamma_{cr}(G) + 2c = p$.

Theorem 3.13

If G = (σ, μ) is a fuzzy cycle with equal fuzzy vertex cardinality, then $\gamma_{cr}(G) = \gamma_{nc}(G)$.

Proof:

Let G = (σ, μ) be a fuzzy cycle graph with vertex set V = { $v_1,v_2,\ldots.v_i,v_{i+1},\ldots,v_n$ } such that v_i is adjacent with $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$ $1 \le i \le n$. Moreover, v_i dominates $v_{(i-1) \bmod n}$ and $v_{(i+1) \bmod n}$. Let $D_{cr}(G)$ and $D_{nc}(G)$ be the fuzzy clique regular dominating set, fuzzy neighborhood clique dominating set respectively. By definition fuzzy clique regular dominating set, fuzzy neighborhood clique dominating set, < $N(D_{cr}(G))$ > is regular and < $N(D_{nc}(G))$ > is complete. Since the complete fuzzy graph with two vertices of equal fuzzy cardinality satisfies the above both conditions, therefore $D_{cr}(G) = D_{nc}(G) = \{ v_1, v_2, \ldots. v_i, v_{i+1}, \ldots, v_n = v_0 \}$ and $\gamma_{cr}(G) = \gamma_{nc}(G)$.

Corollary 3.14

If $\gamma_{cr}(G) = \gamma_{nc}(G)$ then G = (σ, μ) is a fuzzy cycle.

Theorem 3.15

If G = (σ, μ) is a k- regular fuzzy graph with k = 1,2 and $\sigma(v_i)$ = c (constant) for every $v_i \in V$ then $\gamma_{cr}(G) = \gamma_{nc}(G)$.

Proof:

Let G = (σ, μ) be a k- regular fuzzy graph with equal fuzzy vertex cardinality . $D_{cr}(G)$ and $D_{cr}(G)$ are the fuzzy clique regular dominating set, fuzzy neighborhood clique dominating set respectively. $D_{cr}(G) = D_{nc}(G)$ for k- regular fuzzy graph with k = 1,2. Therefore, $\gamma_{cr}(G) = \gamma_{nc}(G)$, since $\sigma(v_i)$ = c, for every $v_i \in V$.

Theorem 3.16

If G = (σ, μ) is K_n o K_1 and $\sigma(v_i)$ = c (constant) for every $v_i \in V$ then $\gamma_{cr}(G)$ - set and $\gamma_{nc}(G)$ - set has all fuzzy pendent vertices.

Proof:

Let G = (σ, μ) be a fuzzy Carona K_n o K_1 with fuzzy vertex set V = { $v_1, v_2, \ldots. v_n, v_{n+1}, \ldots, v_{2n}$ } and $\sigma(v_i)$ = c, for every $v_i \in G$. Let $D_{cr}(G)$ and $D_{nc}(G)$ be the fuzzy clique regular dominating set, fuzzy neighborhood clique dominating set respectively. $D_{cr}(G) = D_{nc}(G)$ = { v_i / v_i is a fuzzy pendent vertex } such that < $N(D_{cr}(G))$ > is regular and < $N(D_{nc}(G))$ > is complete. Therefore, $\gamma_{cr}(G)$ - set and $\gamma_{nc}(G)$ - set consists only the fuzzy pendent vertices.

Theorem 3.17

If G = (σ, μ) is K_n o K_1 all effective edges, then $\gamma_{nc}(G) = \sum_{n=1}^{n} \sigma(vi)$ where v_i's are the fuzzy pendent vertices of G.

Proof:

Let G = (σ, μ) be a fuzzy Carona K_n o K_1 with fuzzy vertex set V = { $v_1, v_2, \ldots. v_n, v_{n+1}, \ldots, v_{2n}$ }. Let $D_{nc}(G)$ be the fuzzy neighborhood clique dominating set, $D_{nc}(G)$ = { v_i / v_i's are fuzzy pendent vertices } such that < $N(D_{nc}(G))$ > is complete. Further, the fuzzy neighborhood clique domination number $\gamma_{nc}(G)$ is the sum of all fuzzy cardinality of the pendent vertices, that is

$\gamma_{nc}(G) = \sum_{n=1}^{n} \sigma(vi)$ where vi's are fuzzy pendent vertices of G.

Theorem 3.18

If G = (σ, μ) is K_n o K_1 has a equal fuzzy vertex cardinality, then $\gamma_{cr}(G) = n\sigma(v_i)$.

Proof:

Let G = (σ, μ) be a fuzzy Carona K_n o K_1 with fuzzy vertex set V = { $v_1, v_2, \ldots. v_n, v_{n+1}, \ldots, v_{2n}$ } and $\sigma(v_i)$'s are equal. $D_{cr}(G)$ is the fuzzy clique regular dominating set such that < $N(D_{rc}(G))$ > is complete, that is V = { v_i / v_i's are fuzzy pendent vertices }.Therefore, $\gamma_{cr}(G) = n\sigma(v_i)$.

Theorem 3.19

If G = (σ, μ) is C_n o K_1 has a equal fuzzy vertex cardinality with effective edges, then $\gamma_{cr}(G) = n\sigma(v_i)$.

Proof:

Let G = (σ, μ) be a fuzzy carona C_n o K_1 with vertices having equal fuzzy cardinality. The fuzzy clique regular dominating set $D_{nc}(G)$ = { v_i / v_i's are fuzzy pendent vertices }.Therefore, $\gamma_{cr}(G) = n\sigma(v_i)$.

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